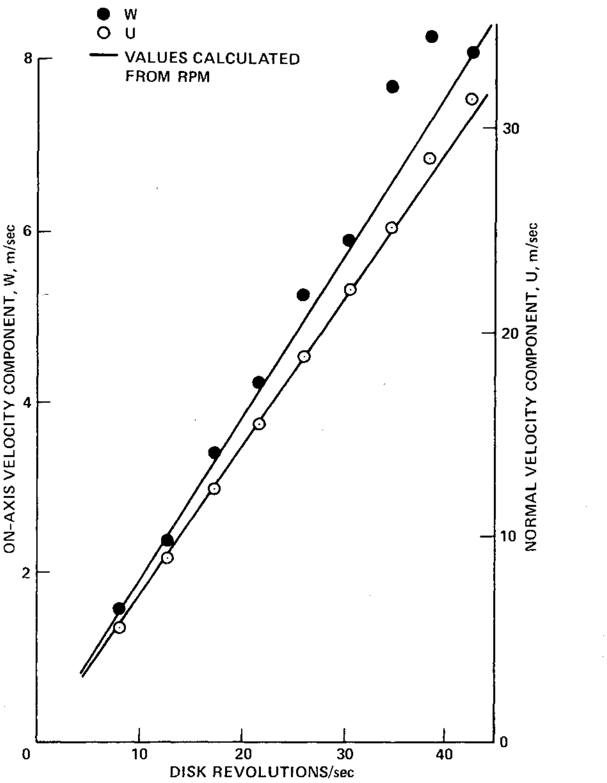

Fig. 3 Measured and calculated values of on-axis and normal velocity components using a rotating disk.

and U is the velocity component normal to the optical axis. If $(W/U)^2 \tan^2(\phi/2) \ll 1$, this equation reduces to

$$\mathrm{d}W/W \approx (1/\sqrt{2})(\mathrm{d}V/V)[(W/U)\tan(\phi/2)]^{-1} \quad (3)$$

Figure 2 shows the error in W using Eq. (3) plotted as a function of W/U for various values of ϕ, assuming a 1% accuracy in measuring V_1 and V_2.

Simulated velocity measurements with a rotating Plexiglas disk were made using the configuration just described. A beam from a 50-mW He-Ne laser was passed through a set of beam splitters to produce four beams, as shown in Fig. 1. A polarization rotator was used to make the polarization of the two pairs of beams orthogonal. The beams were focused to a point with an f/2.5 lens with a focal length of 305 mm. The light scattered by the rotating disk was collected in forward scatter by another lens, focused through an aperture, and passed through a Wollaston prism to separate the scattered light into two beams of orthogonal linear polarization. The light corresponding to the components V_1 and V_2 was detected by two photomultiplier tubes. The outputs of the photomultiplier tubes were analyzed using a spectrum analyzer to obtain the components V_1 and V_2.

Results

The values obtained for the normal and on-axis components U and W are plotted in Fig. 3. In this measurement, the angle between components ϕ was 15 deg and θ was 3.5 deg. The disk was at an angle of 75.25 deg to the optical axis, giving a W/U ratio equal to 26%. The measured velocities are in good agreement with the velocities calculated from the rpm of the disk. The largest errors in W are less than 19%, and the mean error is less than 6%. From Fig. 2, the maximum uncertainty expected for the preceding values of ϕ and W/U, with a 1% accuracy in determing V_1 and V_2, is about 21%. Thus, the measured values are seen to be well within the expected limits.

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Coupled Thermomechanical Effects in High Solids Propellant

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I. Introduction

It has been suggested that the master relaxation modulus together with the time temperature shift factor characterized under isothermal conditions might be used to predict the transient response, i.e., stretching while cooling (or heating), for thermorheologically simple viscoelastic materials.[1] Observations have shown that the extension of linear viscoelastic theory for thermorheologically simple materials will predict a transient stress response which is much lower than that obtained experimentally in modern high solids propellant.[2]

The permanent memory constitutive relation developed from mechanical and thermodynamic theory[3] provides the

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basic formulation for thermomechanical coupled analysis, and suggests the necessary approaches to modify the transient predictions. However, there are two possibilities to modify the transient prediction; one is considered as the modified reduced time approach, and the other the modified modulus approach. The traditional transient formulation[4] is just a special case of the modified formulation just mentioned.

II. Nonisothermal Constitutive Theory

The uniaxial stress constitutive equation for linear viscoelastic materials, developed from mechanical and thermodynamic theory, can be represented by[3]

$$\sigma(t)=\int_0^t \phi_1(T,t-\tau)\dot{\epsilon}(\tau)\mathrm{d}\tau+\int_0^t \phi_2(T,t-\tau)\dot{T}(\tau)\mathrm{d}\tau \tag{1}$$

where ϕ_1 is the linear relaxation modulus, ϕ_2 the thermal relaxation function, ϵ the uniaxial strain, and T the temperature excursion from a rest state. For thermorheologically simple materials, Eq. (1) can be written as

$$\sigma(t)=\int_0^t \phi_1(\xi-\xi')\dot{\epsilon}(\tau)\mathrm{d}\tau+\int_0^t \phi_2(\xi-\xi')\dot{T}(\tau)\mathrm{d}\tau \tag{2}$$

with

$$\xi(t)=\int_0^t \frac{\mathrm{d}t'}{a_T[T(t')]} \tag{3a}$$

$$\xi'(\tau)=\int_0^\tau \frac{\mathrm{d}t'}{a_T[T(t')]} \tag{3b}$$

where a_T is the time temperature shift factor. The relation between ϕ_2 and ϕ_1 is given by

$$\phi_2(t)=-\alpha\phi_1(t) \tag{4}$$

where α is the linear thermal expansion coefficient which is herein considered to be constant. Then the constitutive Eq. (2) will be reduced to the traditional form[4]

$$\sigma(t)=\int_0^t \phi_1(\xi-\xi')\frac{\mathrm{d}}{\mathrm{d}\tau}[\epsilon(\tau)-\alpha T(\tau)]\mathrm{d}\tau \tag{5}$$

But Eq. (4) is not, in general, necessarily true; therefore Eq. (5) can not be expected to give a satisfactory transient prediction.

Modified Reduced Time Approach

The observation results, in stretching while cooling, indicated that the stress response is much larger than the prediction of Eq. (5); therefore a larger (or smaller) relaxation modulus than in the isothermal case should be used to account for the coupled thermomechanical effects. Equations (4) and (5) may be usED to predict the transient response if we change the reduced time to a modified redUced time $\bar{\xi}(t)$ by

$$\bar{\xi}(t)=\xi(t)+\Delta\xi(t) \tag{6}$$

where $\xi(t)$ is the same as defined in Eq. (3), and

$$\Delta\xi(t)=\int_0^t \frac{\mathrm{d}t'}{\bar{a}[T,\dot{T}]} \tag{7}$$

is the change in reduced time due to the transient effect. $\bar{a}[T,\dot{T}]$ is considered as the additional shift factor which provides the thermal coupled effects. Then the constitutive equation represented by Eq. (2), should be changed to the modified form

$$\sigma(t)=\int_0^t \phi_1(\bar{\xi}-\bar{\xi}')[\dot{\epsilon}(\tau)-\alpha\dot{T}(\tau)]\mathrm{d}\tau \tag{8}$$

The characterization of $\bar{a}(T,\dot{T})$ needs some transient experimental data together with the isothermal relaxation modulus.

Modified Modulus Approach

When a coupled thermomechanical effect exists, the thermal relaxation function ϕ_2 in Eq. (2) will not necessarily follow a similar relaxation pattern as the relaxation modulus ϕ_1. Thus, we may assume that ϕ_2 can be represented by

$$\phi_2(T,t)=-\alpha\phi_1(T,t)[1+f(T)] \tag{9}$$

where $f(T)$ is a function of temperature, which together with ϕ_2 will represent the coupled effects. The uniaxial constitutive Eq. (2) should be changed into

$$\sigma(t)=\int_0^t \phi_1(\xi-\xi')[\dot{\epsilon}(\tau)-\alpha\dot{T}(\tau)]\mathrm{d}\tau - \int_0^t \alpha\phi_1(\xi-\xi')f(T)\dot{T}(\tau)\mathrm{d}\tau \tag{10}$$

where the reduced time $\xi(t)$ is the same as defined in Eq. (3). The temperature function $f(T)$ can be approximated by using the isothermal characterized relaxation modulus with some transient experimental data.

III. Numerical Example

Using the relaxation and transient experimental data[2] presented in Ref. 3, approximate the relaxation modulus (E_{rel}) and the time temperature shift factor a_T by

$$E_{\mathrm{rel}}(t)=E_0+E_1 t^{-1/2} \tag{11a}$$

$$a_T=e^{b-KT} \tag{11b}$$

where E_1, E_0, b, and K are constants to be determined by fitting the relaxation data[3] with Eq. (11).

Modified Reduced Time Approach

Assume the additional shift factor $\bar{a}$ has the form

$$\bar{a}[T(t),\dot{T}]=\frac{1}{A\dot{T}}e^{-KT} \tag{12}$$

where A is another constant to be determined. Substitute Eq. (12) into Eqs. (6) and (7), together with Eq. (11); then Eq. (8) has a closed-form solution,

$$\sigma(t)=(R-\alpha k)\left[E_0 t-\frac{2E_1}{Kk}\sqrt{\frac{|Kk|e^{(b-KT_0)}}{|1+Ake^{b}|}}\, e^{-Kkt/2}\tan^{-1}\sqrt{|e^{-Kkt}-1|}\right] \tag{13}$$

where R and k are the time rate of change of strain and temperature, respectively. The constant A is determined by fitting the transient experimental data[3] with Eq. (13). Then the transient and relaxation predictions[3] are shown by triangles in Fig. 1.

Modified Modulus Approach

Approximate the temperature function $f(T)$ in Eq. (9) by

$$F(T)=\alpha f(T)=a+cT \tag{14}$$

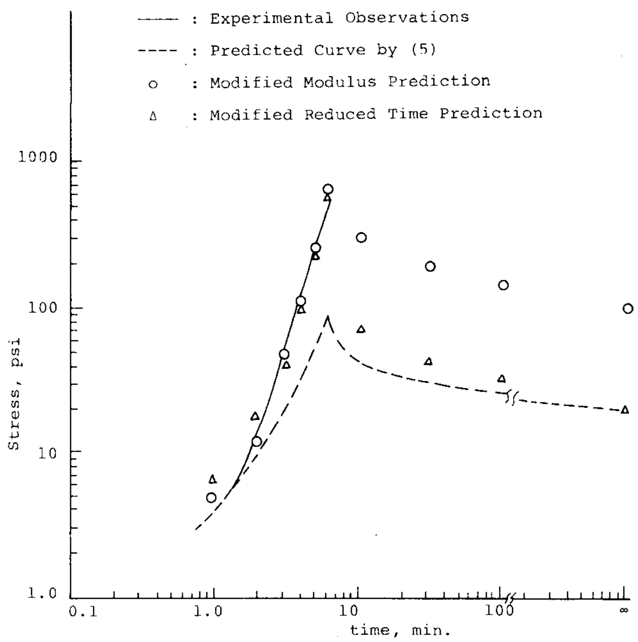

Fig. 1 Comparison of observation and prediction.

where a and c are constants to be determined. Apply Eqs. (3), (11), and (14) with Eq. (10); the stress response in the transient portion is

$$\sigma(t) = (R-\alpha k)\left[E_0 t - \frac{2E_I}{Kk}\sqrt{|Kk|e^{(b-KT_0)}}\right.$$

$$\left. e^{-Kkt/2}\tan^{-1}\sqrt{|e^{-Kkt}-1|}\right] + \Delta\sigma(t) \tag{15}$$

where

$$\Delta\sigma(t) = af_1(t) + cf_2(t) \tag{16}$$

$$f_1(t) = -E_0 kt + \frac{2E_I}{K}\sqrt{|Kk|e^{(b-KT_0)}}\, e^{-Kkt/2}$$

$$\times\tan^{-1}\sqrt{|e^{-Kkt}-1|} \tag{17}$$

$$f_2(t) = -E_0 T_0 kt + \frac{E_0 k^2}{2}t^2 + \frac{2E_I}{K}\sqrt{|Kk|e^{(b-KT_0)}}$$

$$e^{-Kkt/2}\left[T_0\tan^{-1}\sqrt{|e^{-Kkt}-1|}\right.$$

$$\left. + k\int_0^t \tan^{-1}\sqrt{|e^{-Kk(t-\tau)}-1|}\,\mathrm{d}\tau\right] \tag{18}$$

substitute the transient experimental data[3] into Eq. (15), through numerical calculations; the constants a and c for these materials are determined by using a least-square technique.[3] Then the transient and relaxation predictions by this approach are shown by circles in Fig. 1.

IV. Conclusions

When the predicted results are compared with observations, the following conclusions can be drawn.

1) The transient prediction by Eq. (5) will lead to a large error due to the coupled thermomechanical effects.

2) The modified reduced time approach predicts a consistent result with observations in the transient portion. But the prediction in the relaxation portion is questionable, since it predicts the same equilibrium value as predicted by Eq. (5).

3) The modified modulus approach will predict consistent results in both the transient and relaxation portions. Therefore, it may be considered to be the proper form to represent the coupled thermomechanical effects for highly filled solid propellants.

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Optimum Design of Laminated Plates under Axial Compression

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Introduction

RECENTLY, filamentary composite materials have been suggested for the primary structure of aircraft and spacecraft. The reason is primarily the weight savings that can be attained. Many design criteria must be considered in the design of composite materials for structures; the buckling criterion is one. Several theoretical and experimental papers have been published on the buckling of laminated composite plates under axial compression. Housner and Stein[1] calculated the optimum fiber direction of graphite-epoxy sandwich panels under axial compression, assuming that the angle of all the plies is the same.

This paper presents a method to design laminated plates with orthotropic layers under uniaxial and biaxial compression. The design criterion is the buckling stress. Each layer of the plate is assumed to have the same thickness and an equal number of fibers of the same kind in the $+\alpha_i$ and $-\alpha_i$ directions with respect to the x coordinate in the same type of matrix. Therefore, each layer can be considered to be orthotropic. Inhomogeneity in the direction of the thickness of the plate (stacking sequence) is taken into account in the calculation; therefore, the present calculation allows almost complete freedom for the choice of all the ply angles.

The present problem is to find the fiber directions of all the layers that give the highest buckling stress; therefore, one has to solve an unconstrained maximization problem. The objective function and the design variables are the critical buckling stress and the fiber directions, respectively. Preassigned parameters are the material properties, the thickness of each layer, the number of layers, the aspect ratio of the plates, and the ratio of the force per unit width in the y

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